

## LONG-PERIOD WEATHER CHANGES AND METHODS OF FORECASTING

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## I. BASES OF FORECASTS

Two distinct periodic changes are recognized in meteorology: One a daily, the other an annual, period. These are clearly related to the changing position of the sun. In addition to these daily and annual periods there are series of apparently irregular changes which are called the weather.

The irregular changes from day to day are found to be associated with centers of high and low pressure, and with air masses of different origins. The variations shown by weekly, monthly, or annual means are thought by some meteorologists to be the unbalanced means of daily changes and hence purely fortuitous. Others like myself believe them to be the results of large atmospheric movements of orderly procedure, and hence predictable when understood.

When the annual and daily periods are eliminated and the departures of many stations are plotted on maps over a large area, as for example the United States, it is well known that there appear distinct areas of plus and minus departures, usually with well defined centers showing maximum departures. These areas are generally several hundred miles in extent. Henryk Arctowski has proposed the names *meion* and *pleion* for them; these names are derived from the same Greek roots (viz, *meion*, less, and *pleion*, more) as the geological terms *Miocene* and *Pliocene* (variants of which are *Meiocene* and *Pleiocene*), usually pronounced "my-o-seen" and "ply-o-seen", respectively. The names are easily remembered, since *meion* signifies an area of minus departures, and *pleion* an area of plus departures. They are purely descriptive and involve no theory of their origin and hence will probably be acceptable to all parties. Whether *meiopleion* will be acceptable as a general term involving both areas is less certain.

These departures from normal may be for different intervals of time as, for example, departures of daily means from normal, departures of weekly means, monthly means, annual means, or the means of other intervals of time. Hence, to describe fully such *meions* and *pleions* a statement of the interval of time is needed so that there would occur such expressions as daily *pleions*, weekly *pleions*, monthly *pleions*, annual *pleions*, 11-year *pleions*, etc.

The departures for different elements may be designated by suffixes, as, for example, *baropleions* for pressure departures, *thermopleions* for temperatures, departures, and *ombropleions* for departures of rainfall from normal.

In describing the process of smoothing in part III, I also suggest a new term, namely, *harmonics* for data smoothed by harmonic terms rather than by the ordinary numerical process.

My researches on weather changes of long period began more than 50 years ago and my present views can best be explained by the step-by-step development of my researches. In my first study I eliminated the annual period in pressure at a number of stations in the United States for the period 1874 to 1881 by taking the means of every 12 months, adding 1 month and dropping 1 month progressively. Such means have been variously called progressive means, overlapping means, moving means, running means, or chain means.

When such means for 16 widely separated stations in the United States were plotted in curves and on maps they showed several important facts:

(1) There were marked oscillations in pressure about 25 months in length, during the period covered by the observations. These oscillations were combined with a longer oscillation, which was separated from the shorter period by getting moving means of 25 months. These are shown by the graphs in figure 1. The continuous curves are the plots of the 12 monthly means and the broken curves are the means of the 25 months for four stations.

(2) The 25-month period was separated from the longer period by subtracting the 25-month moving means from the 12-month moving means. When the departures were plotted on maps they showed distinct centers of plus and minus departure separated by intervals of several hundred miles. In other words, when the pressure was above normal in one area it was below normal in some distant area. This condition is illustrated in figure 2.

(3) There was a seesaw oscillation between these areas, but in each successive return the centers of oscillation

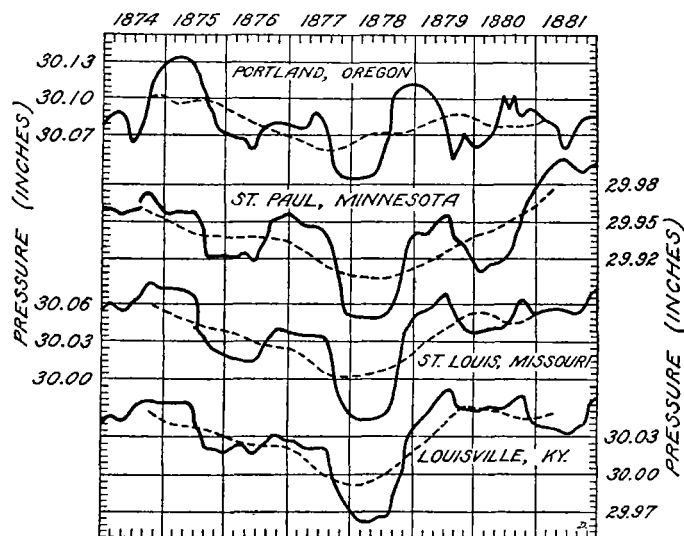


FIGURE 1.—Twelve-month means of pressure, showing oscillations of pressure of slightly over 2-year duration (see plot for St. Paul).

were displaced, showing an irregular but progressive movement. This movement is indicated by the line containing small circles in the bottom chart of figure 2. Each circle shows the progressive position of the center of maximum oscillation, whether minus or plus. The first mental picture derived from this condition was that the air over the continent grew alternately denser and rarer, such as the air might do over a vibrating metal disk; but instead of the nodes being fixed as in the case of the disk, the nodes were in movement because of irregularity in the forces causing the vibration. Later investigation showed, however, that the picture should be more like that of a disk across which waves were in progress but did not arrive at the same position after equal intervals of time because of irregularities in their velocity or direction.

When the monthly rainfalls for various sections in the interior of the United States were smoothed by 12 monthly means in the same way as the pressure, there appeared similar oscillations of about 25 months with maxima of rainfall at the time of minima of pressure.

The striking contrast between the winter temperatures during the winters when the pressure was high and those when the pressure was low is shown by the following

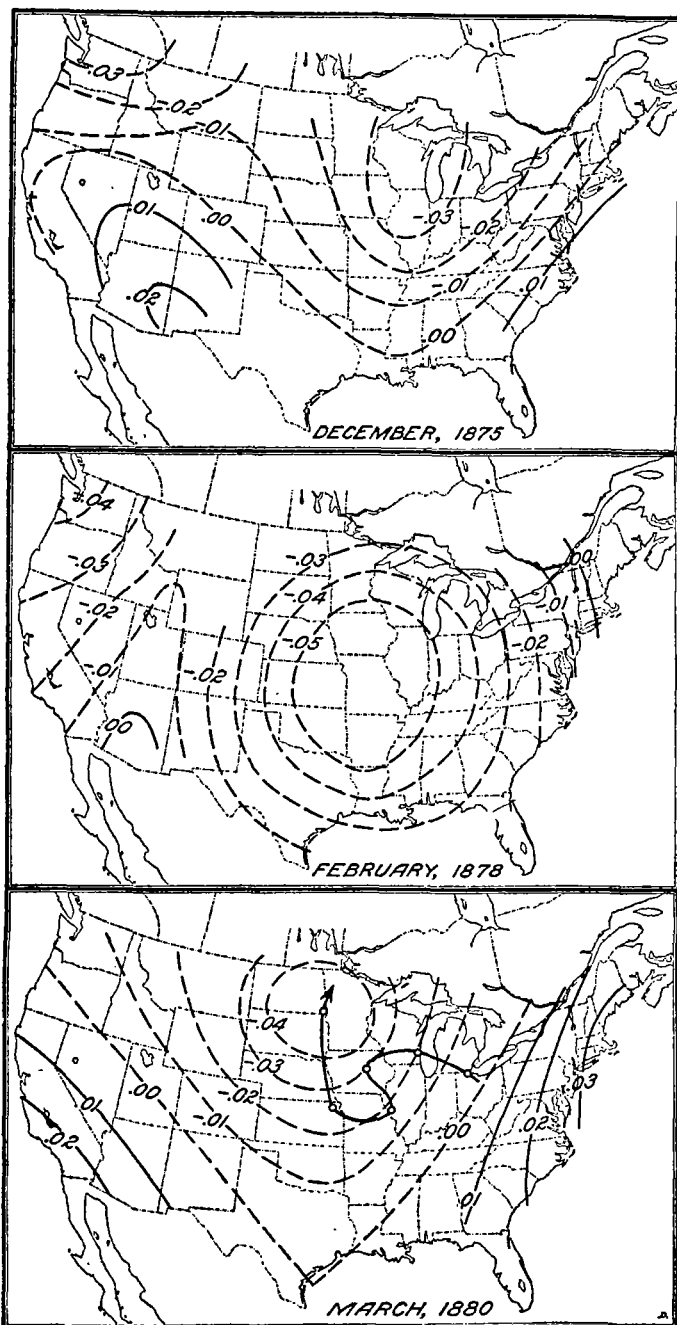


FIGURE 2.—Centers of greatest minus departure, in period of slightly over 2 years, showing movement of the center of oscillation.

departures from normal temperature taken from Dunwoody's *Signal Service Table of Rainfall and Temperature*:

	1874-75	1875-76	1876-77	1877-78	1878-79	1879-80
Upper Lake region.....	-21.9	+12.4	-0.3	+32.0	-1.2	+13.9
Ohio Valley and Tennessee.....	-8.3	+22.1	-3.8	+17.2	-7.6	+25.1
West Gulf States.....	-1.3	+16.1	-7.9	+3.6	-7.0	+19.1
South Atlantic States.....	-0.1	+11.1	-9.0	+3.6	-4.0	+27.6

(4) Since the centers of oscillation were in movement, the phases of the oscillations were reversed in certain regions, as, for example, at stations in the eastern United

States. (See fig. 2.) Hence, it would be difficult if not impossible to determine periodic changes in atmospheric conditions at any one station or group of stations in regions subject to such changes of phase.

The results of this investigation were published in the *American Meteorological Journal* for August 1884 and April 1885, volume 1, pages 130 and 528.

The next step in my investigation was to eliminate the annual and diurnal periods from observed data by getting means for many years and subtracting these mean values from the observations to obtain the irregular departures called weather. It was found that these variations under successive smoothings fell into a definite number of oscillations of longer and longer periods, from a few days to many years or even centuries. A study of these complex oscillations was given in the *MONTHLY WEATHER REVIEW* for April 1907 and in my book, *World Weather*, 1923, pages 111-123, 128-130, and 135-136. Examples were given of the different behavior of oscillations of different lengths and hence of the probability that they were real phenomena of our atmosphere and not merely accidental variations from mean values.

In order to illustrate these processes of smoothing in this paper the mean monthly departures from normal temperature at Chicago were successively smoothed by moving means of 5 months, 7 months, 11 months, 15 months, and 21 months, and the means placed in the middle of the interval of time covered. The results are plotted in figure 3.

It is seen that the curves are irregular; but in the smoothed values for 5 and 7 months there occur distinct maxima *a*, *b*, *c*, etc. It is evident that these are real maxima of pressure and not maxima created by the process of smoothing, because different degrees of smoothing by three, five, or seven do not change the position of the maxima. They do not disappear until the number of smoothing terms is near the length of the oscillation.

These maxima marked *a*, *b*, *c*, etc., are about 11 to 13 months apart and disappear when moving means of 11 months are obtained. In the moving means of 11 months other maxima appear which are indicated by the letters *A*, *B*, *C*, etc. These maxima are about 25 to 35 months apart and mostly disappear when moving means of 21 months are obtained. Then other maxima marked *I*, *II*, etc., appear. In other words, by means of successive smoothing, meteorological changes can be separated into a number of distinct oscillations of different lengths.

Different processes of smoothing have been studied in the development of this work and are described in part III. As a result of this study, smoothing by harmonic terms was considered the best, for the reasons set forth, and is the process of analysis now used in preparing my material for study and for forecasting. The smoothing of monthly data harmonically as now practiced by me is shown in figure 4. In examining this figure it should be noted that a 12-month oscillation is best brought out arithmetically by moving means of 6 months, but in harmonic smoothing a 12-month oscillation is best shown by smoothing with 12 terms.

When data for a network of stations are worked up in the manner shown in figure 4 and plotted on maps such as are illustrated in figure 2, the areas of *minus* and *plus* departures which we will call *meions* and *pleions* move with different velocities corresponding more or less inversely with the length of the period of oscillation. In other words, the *meions* and *pleions* of longer periods move more slowly, and frequently in different directions from those of shorter period prevailing at the same time. The shorter *meioleions* usually move from west to east in the United

States, and only occasionally from other directions; while those of longer periods may move from any direction, sometimes from west to east or from north to south and sometimes in the opposite direction.

By following the movements of these *meions* and *pleions* their position can be estimated, and forecasts based on them, for a limited time in advance. In order to extend these forecasts for a considerable interval in advance, it is necessary to assume some sort of rhythm or regularity of occurrence in succeeding pulsations. This assumption frequently leads to disappointment, owing to changes in direction and velocity of the *meiopleions*. In order to get at the law of these changes it was necessary to ascertain their causes.

Among many efforts in this direction, a comparison was made of the pulsations of pressure and of temperature with pulsations in the values of solar radiation as measured by the Astrophysical Observatory of the Smithsonian Institution. This comparison showed many facts which indicate a close relation between the solar changes and terrestrial weather. These studies were published in the *Smithsonian Miscellaneous Collections*, between the years 1917 and 1934.

One of the early comparisons made in 1916 was that of 10-day moving means of solar radiation with 10-day moving means of temperature at stations in Argentina where I was forecast official of the Argentine Weather Service. A plot of these moving means is reproduced in figure 5. The comparison of these moving means with those of solar radiation 2 days later showed a correlation of  $-0.82$  at Sarmiento, in southern Argentina, from which region the *meions* and *pleions* moved northeastward and took about 8 days to reach southern Brazil. Had the position of origin of the *meiopleions* remained permanent, it would have been easy to anticipate changes of temperature in South America from changes in solar radiation. Unfortunately, however, further comparisons showed that the places of origin of the *meions* and *pleions* were not constant, so that meteorological changes in any given region might be for a while positively correlated with solar changes and a little later negatively correlated with solar changes.

This fact is set forth clearly in a study presented to the American Geophysical Union in 1935. (*Trans. of the Amer. Geoph. Union*, 16th annual meeting, 1935, p. 158.) In this paper a comparison was made between an 11-month period which has been discovered in solar radiation and changes in air pressure at widely separated stations in North America. The comparison was made by means of values of solar radiation and pressure smoothed harmonically with 12 terms (see part III) and further smoothed by taking overlapping means of three periods. The results are shown by plots in figure 6. It is seen from these plots that complete reversal of the pressure oscillations as compared with solar radiation are of frequent occurrence. Moreover the inversions do not occur simultaneously at the different stations. Studies of this effect have led me to believe that it results from changes in the places of origin of the *meions* and *pleions* with changes in the intensity of solar radiation. This shifting greatly complicates the problem. The fact stands out clearly, however, that the pulsations in meteorological conditions are closely related to pulsations in solar radiation, and any periodicity which may be found in solar radiation will be reflected in terrestrial conditions.

In order to ascertain whether there were certain normal positions around which the *meions* and *pleions* oscillated, 12-month *harmonics* of pressure (see part III) were computed for a network of stations over the Northern Hemis-

phere and at scattered stations in the Southern Hemisphere for the 10 years 1921 to 1930. The data were taken from *World Weather Records*, 1921-30; *Smithsonian Miscellaneous Collections*, volume 90, 1934. During this period there were 10 distinct pulsations of solar radiation at fairly regular intervals of about 11 months. The meteorological pulsations were much less regular, but were averaged for the 10 periods at the various stations, with the hope of thereby obtaining an approximation to normal. These means were plotted on maps for each month of the period. Two of these maps are reproduced, figure 7 showing the mean position of the *meions* and *pleions* at the time of maximum solar radiation in the 11-month period, and figure 8 showing the mean position at the time of minimum solar radiation. These figures show very clearly the influence of oceans and continents. In general, with a maximum of solar radiation, diminished pressures are found over the warm waters of the tropics and over the northern continental masses; while with a minimum of solar radiation, the reverse is found.

When maps similar to figures 7 and 8 are examined for each month of the period it is found that the *meions* and *pleions* are slowly displaced and the centers of greatest departures progress along definite tracks until they disappear. The centers in the Pacific marked + and - in figures 7 and 8 moved southeastward and after about 5 months disappeared near the American coast. The centers in the western United States moved northward and disappeared in Alaska. The centers over the Atlantic near the east coast of the United States moved eastward to the waters near the north coast of Africa and the centers in central Siberia moved westward to the North Atlantic Ocean.

The progress of these *meions* and *pleions* during the 11-month period is most easily shown by curves as in figures 9 and 10. The top curve in each case is derived from the mean values of solar radiation for each month of the period. The lower curves in each case are derived from the mean values of pressure for each month of the period at stations along the tracks of the moving *meions* and *pleions*.

It is seen from figure 9 that in the central Pacific at  $50^{\circ}$  N. and  $170^{\circ}$  W. the pressure rose and fell simultaneously with the rise and fall of solar radiation, but at places further east the maxima and minima were successively delayed until the coast of California is reached. On the other hand, over the subtropical arid lands of northern Mexico and the southern United States the pressure fell and rose in opposition to the rise and fall of solar radiation; that is, the pressure fell over these regions as the sun became hotter. The maxima and minima of pressure were delayed at stations further north and did not reach Alaska until 4 or 5 months later. Over the Atlantic Ocean and Asia a similar series of events occurred (fig. 10). Over the western Atlantic the pressure rose and fell in unison with the solar radiation, and the maxima and minima of pressure occurred successively later at points further east until near the north coast of Africa there was a delay of about 5 months. Over the subtropical arid regions of northern Africa and over subtropical Asia the pressure changed oppositely to the solar change, as it did over the subtropical lands of America, and the times of maxima and minima were delayed until they reached the coast of Norway 5 months later.

There is, however, a terrestrial influence on these changes—the pressure oscillations increase in intensity as they move northward, and diminish as they move southward. The explanation of this condition is that the

wind velocities in general increase as one approaches the axis of the earth's rotation at the poles. The increased movement of the wind increases the speed and intensity of the pressure and temperature changes. This result has been explained by Ferrel and others. The changes of intensity are roughly inversely proportional to the cosine of the latitude, and can be computed with sufficient accuracy for practical purposes on that basis.

The results shown in figures 7 and 8 are all consistent with one another. They indicate that with increased solar radiation the areas of high pressure normally over

were then averaged separately. Some of the curves derived from these averages are plotted in figures 11 and 12. They show that the range from maximum to minimum of solar radiation increased from 0.0060 calory to 0.0082 calory, or about 35 percent.

In the Pacific where the pressure rises and falls in unison with the solar radiation the *pleion* is displaced 20° of longitude toward the west with increased solar radiation, that is to 170° east longitude, and also some 10° of latitude toward the north. The result is that the *pleions* are some 3 to 4 months later in reaching given

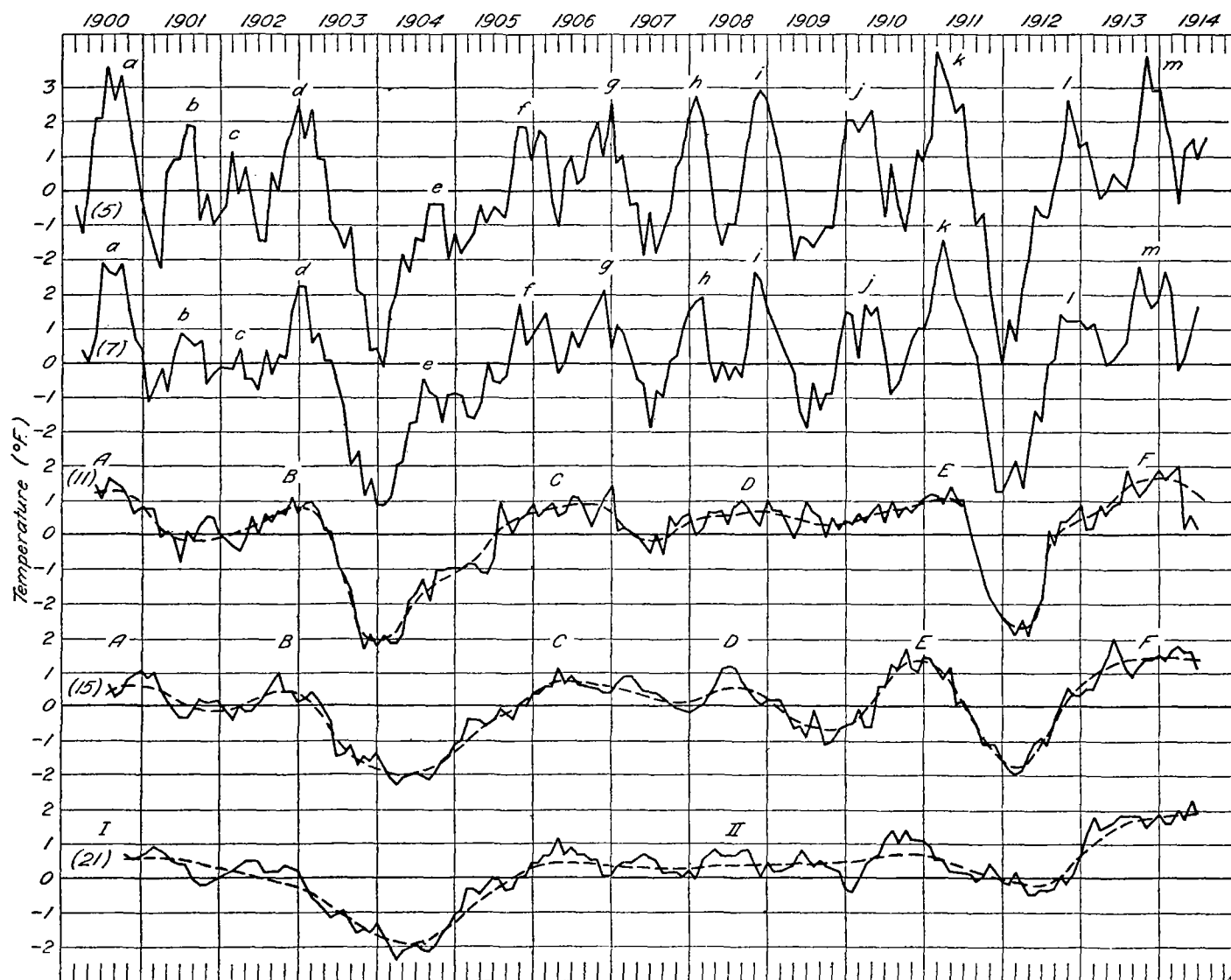


FIGURE 3.—Monthly mean temperature departures at Chicago, smoothed by moving means of 5, 7, 11, 15, and 21.

the oceans in middle latitudes are displaced northward and eastward of their normal positions, and return gradually to their normal positions with decreased solar radiation. Over the subtropical land surfaces the increased heat emitted by the sun causes a fall of pressure and this fall is propagated northward from Helwan to Bodo as shown in figure 10.

When one examines the individual periods, however, he finds that some of them vary materially from the mean; and in order to ascertain the cause of this difference I selected the 3 periods which showed the largest departures from the mean of the 10 periods averaged. The solar radiation and the pressure for these three periods

longitudes farther east. In the mean of all the periods (fig. 9) the maximum of pressure reached the Pacific Coast of the United States in the seventh month of the period. In the mean of three periods with higher solar radiation (fig. 11) the maximum reached the Pacific Coast in the tenth month of the period; that is, 3 months later. In the mean of three periods a *baromeion* formed in northern Mexico and Texas, as shown by the right-hand curves of figure 11, and reached Edmonton in the same months as did the mean of all 10 periods, figure 9. However, in the mean of three periods the *meion* proceeded northward toward the Arctic Ocean instead of north-westward toward Alaska.

Figure 12 shows that over the Atlantic Ocean the *baropleion* in the mean of three periods formed north of the position indicated for the mean of all in figure 9 and moved eastward to Norway. A *baromeion* formed over northern Africa as before and moved northward to Dickson in the Arctic Basin. In other words, the *pleions* over the oceans formed further north and west with increased intensity of solar radiation and were several months later in reaching middle latitudes. The *meions* also followed courses to the eastward of those followed by

took place between Bodo, Norway, and Madeira, as shown by the curves on the right-hand side of figure 13.

In the case of the sunspot period, there is found a similar set of relations with changes in the intensity of solar activity, as is shown by figure 14 taken from my paper on *World Weather and Solar Activity* (Smith. Misc. Coll., vol. 89, no. 15, p. 10, May 1931).

The plots in figure 14 show that there was a seesaw oscillation in the relation of the pressure to sunspots between New York and Upernivik, and evident displace

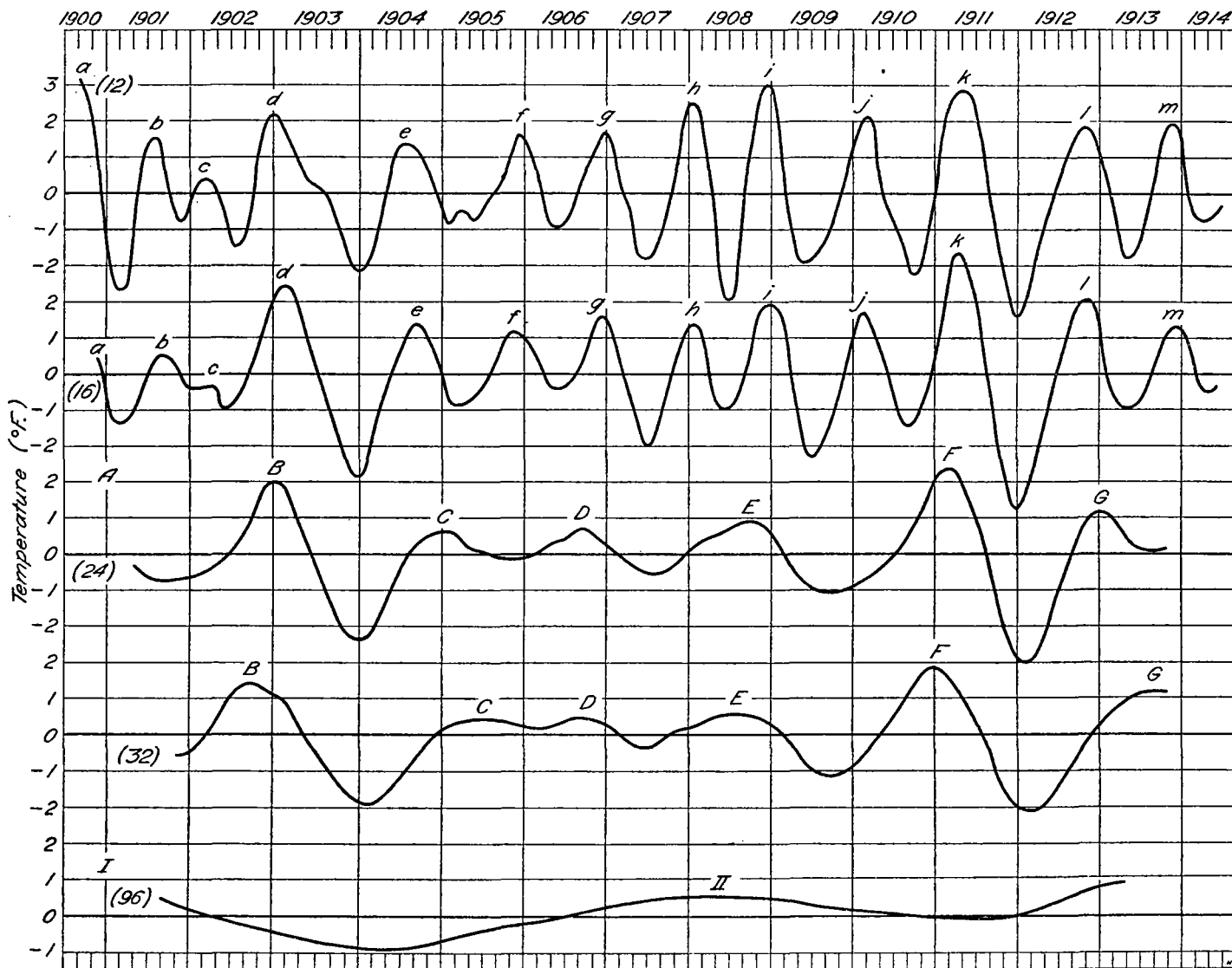


FIGURE 4.—Monthly mean temperature departures at Chicago, smoothed harmonically with 12, 16, 24, 32, and 96 terms.

the *meions* derived from mean values of all the periods and moved more nearly northward.

Another effect of the westward and northward displacement of the *pleions* over the oceans was the inversion of the pressure changes at certain places in high and low latitudes. This fact is made evident by the curves in figure 13. In the means of seven periods of lesser solar radiation shown by the curves marked *a*, the pressure followed the same course as the solar radiation at Kodiak, Alaska, and was inverted at Honolulu, Hawaii. In the mean of three periods with increased range in solar radiation the *pleion* was displaced toward the west and the pressure was inverted to solar radiation at Kodiak and was direct at Honolulu. Inversions of the same kind

ment of the *pleions* northward with increased solar activity. Also, the ranges were greatest in 1917 when the sunspots showed the greatest activity and the solar radiation averaged higher than at any time since the beginning of observations.

In general the temperature averages relatively high at sunspot maximum over the subtropical land surfaces of northern Africa, southwestern United States, and central Australia, as is shown in my book *World Weather*, 1923, page 315.

In order to study further the relation between sunspots and weather, the annual means of sunspots since 1880 were smoothed harmonically with 12 terms, and the annual means of temperature for the same period at numerous

stations in the Northern Hemisphere were smoothed in the same manner.

Plots for a series of stations running from Mexico northward are shown in figure 15, and plots for a series of stations running from Chicago westward to the Pacific Coast are shown in figure 16. It is evident from figure 15 that the *thermopleions* appear in northern Mexico soon after the sunspot maximum and progress northward,

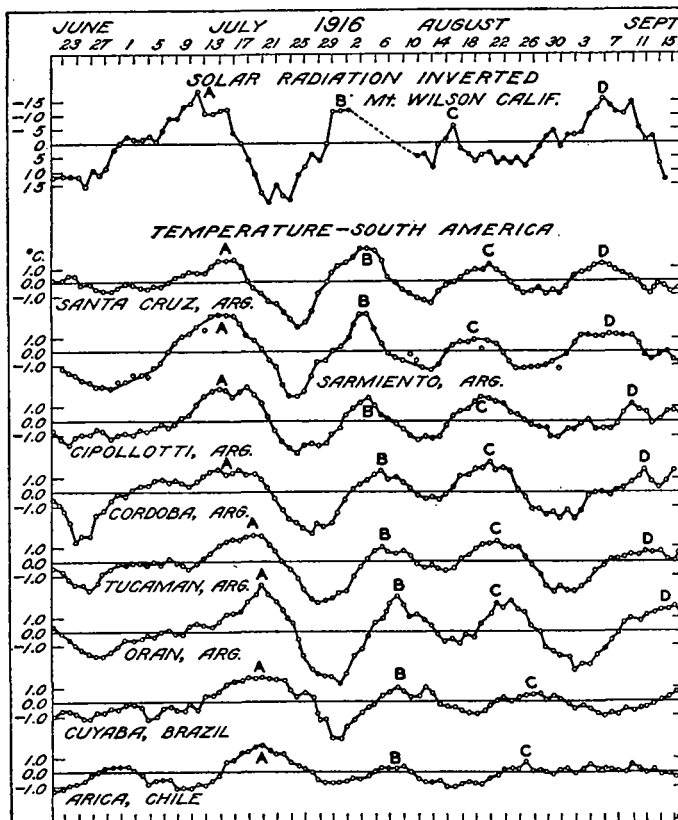


FIGURE 5.—Ten-day means of solar radiation compared with 10-day means of temperature in South America (World Weather, p. 223).

arriving at the northern boundary of the United States about the time of sunspot minima, and reach northern Canada about 3 years later. They thus follow the same course as the *baromeions* of the 11-month period as shown in figure 9.

It is very evident from figure 16 that there was also a westward component of motion from the central United States toward the Pacific coast.

A large array of additional facts in my possession might be given to sustain the views set forth above, but those given seem sufficiently convincing.

It seems clearly evident that there is a close relation of atmospheric changes to periodic changes in solar radiation and also to the sunspot period of about 11 years. The relation, however, is a very complex one owing largely to the change in place of origin of *meions* and *pleions* in the atmosphere with changes in the intensity of solar radiation; but my experience convinces me that we have now sufficiently unraveled the manner in which these changes occur to make useful long-range forecasts.

## II—METHODS OF FORECASTING

The changes in solar intensity might be followed directly from observed values of solar radiation were the observations sufficiently accurate. This, however, is not the

case. Owing to the very great difficulty in freeing the observations of solar radiation from atmospheric interference, the probable errors of these observations at present are nearly as large as the departures from the normal which are being measured. For this reason we cannot well use the day-to-day observations of solar radiation for forecasting weather.

Assuming, however, that the probable error of an individual observation is  $\epsilon = \pm 0.005$  cal. (Abbot now estimates it as somewhat less), the probable error of a month with

25 observations is  $\epsilon_m = \frac{\pm 0.005}{\sqrt{25}} = \pm 0.001$ . This mean error

is several times less than the monthly deviations from normal, so that monthly means can be usefully used for investigation and forecasting. Where smoothed values of 11 months are used, as in the investigations quoted above, the probable error becomes about 18 times smaller than that of the individual observations, and changes of 0.002 calorie can be followed with assurance. This reasoning is on the assumption that there are no constant errors such as might be introduced by volcanic eruptions, etc.

My method of forecasting in use at the present time is to smooth the meteorological data for a network of stations for successive periods of increasing length in the manner illustrated in figure 4. The values are then charted month by month, day by day, week by week, year by year, or other unit of time, and lines of equal deviation drawn. Successive maps show the direction and velocity of the *meions* and *pleions*, and these can be used for forecasting. By studying their velocity and direction of motion, projecting them ahead and combining them, or by detecting the period of recurrence, dating the map one or more periods ahead and combining graphically with other periods similarly treated, forecasts may be made for long intervals in advance.

However, the process which is used in preference at the present time is to smooth the solar data and the meteorological data in the same manner and plot them as curves

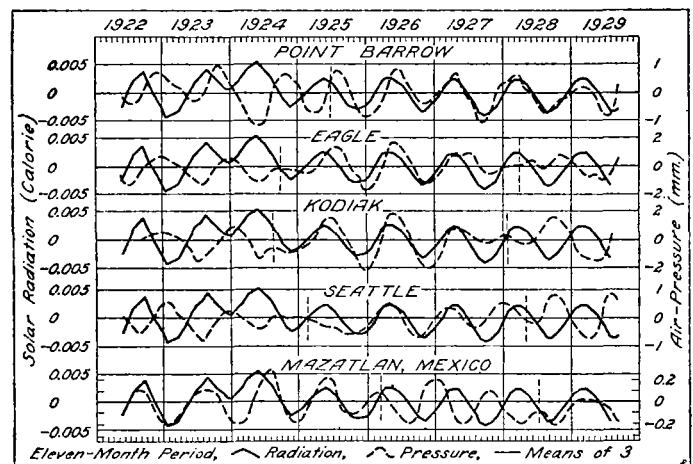


FIGURE 6.—Eleven-month period in solar radiation and in pressure.

over each other in the manner shown in figure 6. Then the relation to the solar period can easily be seen, so that the meteorological data can be extrapolated one or two periods in advance, with only occasional failures on account of changes of phase. In each case the length of a known solar period is used for the extrapolation. This is done for a succession of periods of different lengths, and the forecasted values added together to obtain the expected value at that station. This process is followed for a num-

ber of stations and the results plotted on maps. Lines of equal value are then drawn and the result is a map of expected occurrence, or a forecast, whether it be for pres-

figure 18 shows the actual departures of temperature as published in the *Weekly Weather and Crop Bulletin* of the United States Weather Bureau, June 2, 1936.

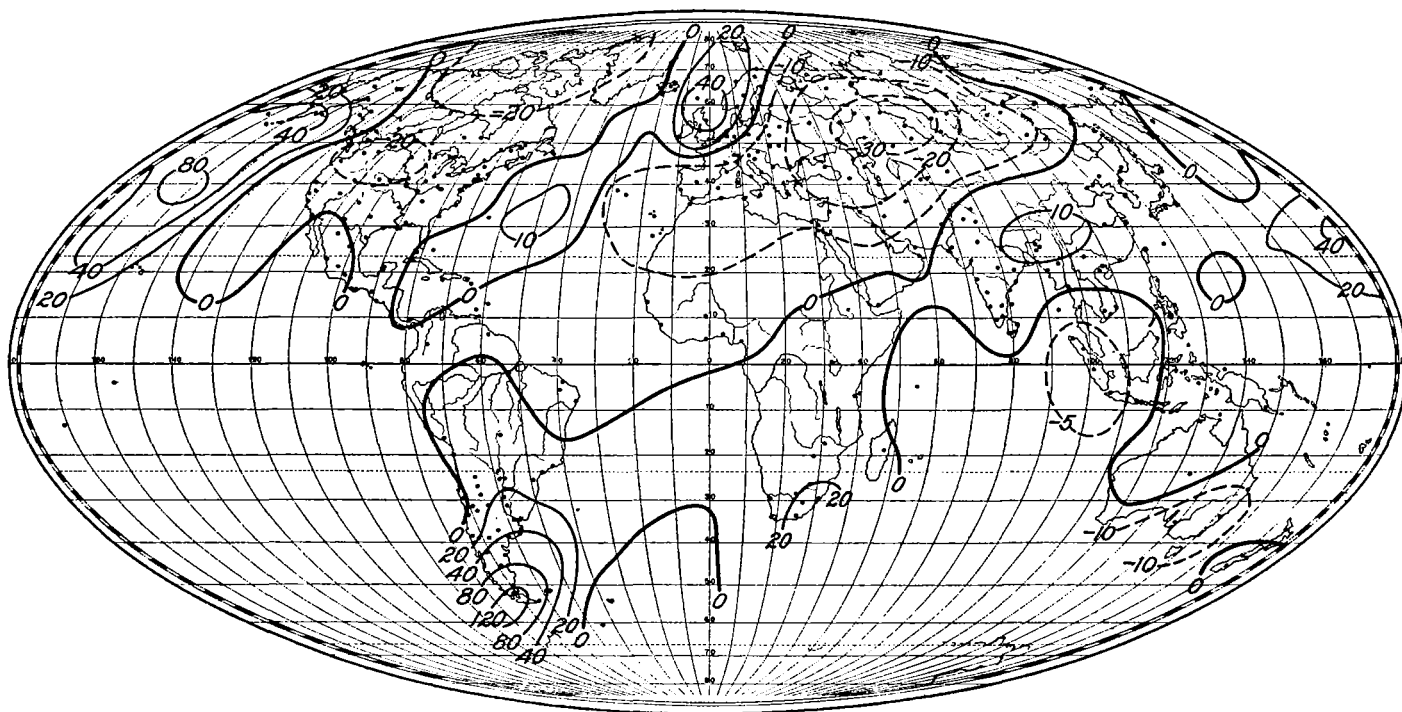


FIGURE 7.—Departure of pressure at time of maximum of solar radiation in 11-month period (in units of 0.01 mm).

sure, temperature, or rainfall. All three are used, whenever possible, and checked against one another by means of the relations known to exist between them.

Weekly forecasts can be made in the same way. Figure 19 shows the forecast map of temperature for the second week in July, made in the latter part of June, outlining



FIGURE 8.—Departure of pressure at time of minimum of solar radiation in 11-month period (in units of 0.01 mm).

A map of temperature departures for May 1936, made in this way during the latter part of April and actually used in forecasting the temperature of May for different sections of the United States, is shown in figure 17, while

the heat wave which occurred east of the Rocky Mountains during that week as shown by the data in the *Weekly Weather and Crop Bulletin* of July 16, 1936, reproduced in figure 20. The intensity of the departures was

not as great in the forecast map as in the observed map, but areas of excess and defect were fully 80 percent correct. Forecasts based on this map were published in my bulletins, as follows: "July promises to average warm in the eastern half of the United States and cool on the Pacific coast and Rocky Mountains. The warmest part

My early investigations of short meteorological periods led me to the belief that these periods were nearly always subdivisions of longer periods (*Science* (N. S.), vol. 7, p. 243, 1898). My first suggestion of their connection with solar periods was in connection with a period of about 7 days which I found to be nearly one-fourth of a solar

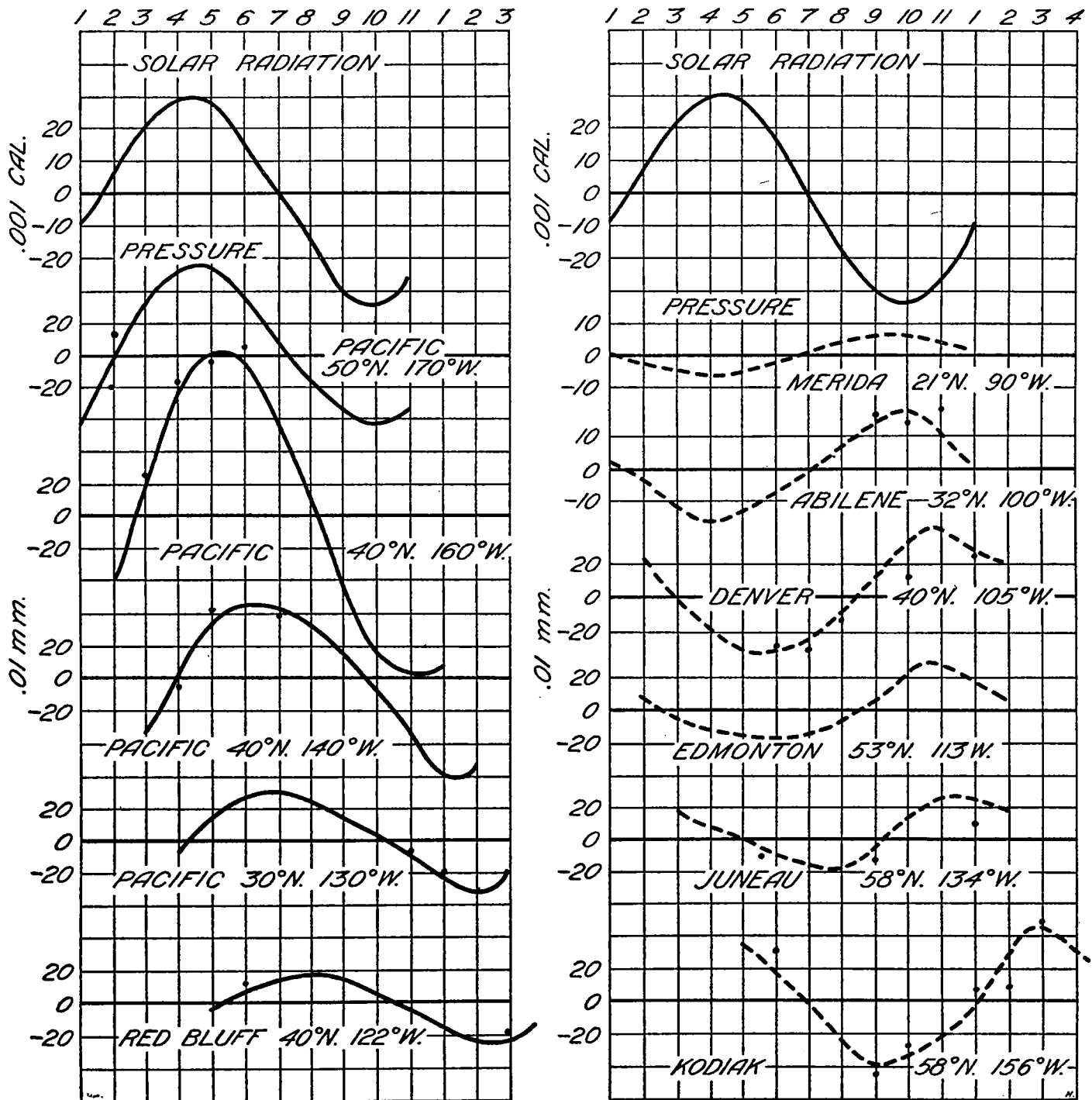


FIGURE 9.—Eleven-month period; means of 10 periods (1).

of the month in eastern sections is indicated between the 1st and 21st." It seems clear from the foregoing that practical long-range forecasts are now being made which compare favorably in accuracy with the day-to-day forecasts of the various weather bureaus of the world. Such forecasts will increase in accuracy as we come to know more exactly the lengths of the solar periods and can predict their changes in amplitude.

rotation, but which was subject to sudden discontinuities (*Amer. Jour. of Science*, New Haven, vol. 2, p. 7, 1898). For longer weather periods, I tried subdivisions of the sunspot period and its multiples (*Nature*, vol. 51, p. 436, 1895).

Later investigations led me to try the double sunspot period of 22.5 years as the fundamental period of solar and weather changes (*Smithsonian Misc. Coll.*, vol. 82,

no. 7, p. 37). This period and its subdivisions  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ , etc., gave me better results than the single sunspot period. I worked out these periods in meteorological changes and made forecasts based on them. In order to eliminate the annual period I took the same month for each successive year and made forecasts for the months separately. One made for June temperatures at New Haven 4 years in advance was sent out to clients in May

shorter periods of solar radiation were subdivisions of this length. He made forecasts of solar radiation based on this supposition, which he finds were approximately verified. He also made forecasts of meteorological changes based on the same periods (*Report on the Astrophysical Observatory*, 1935, Smithsonian Institution, Washington, D. C.).

A. E. Douglass has also been actively at work on this question of periodicity in solar and meteorological changes,

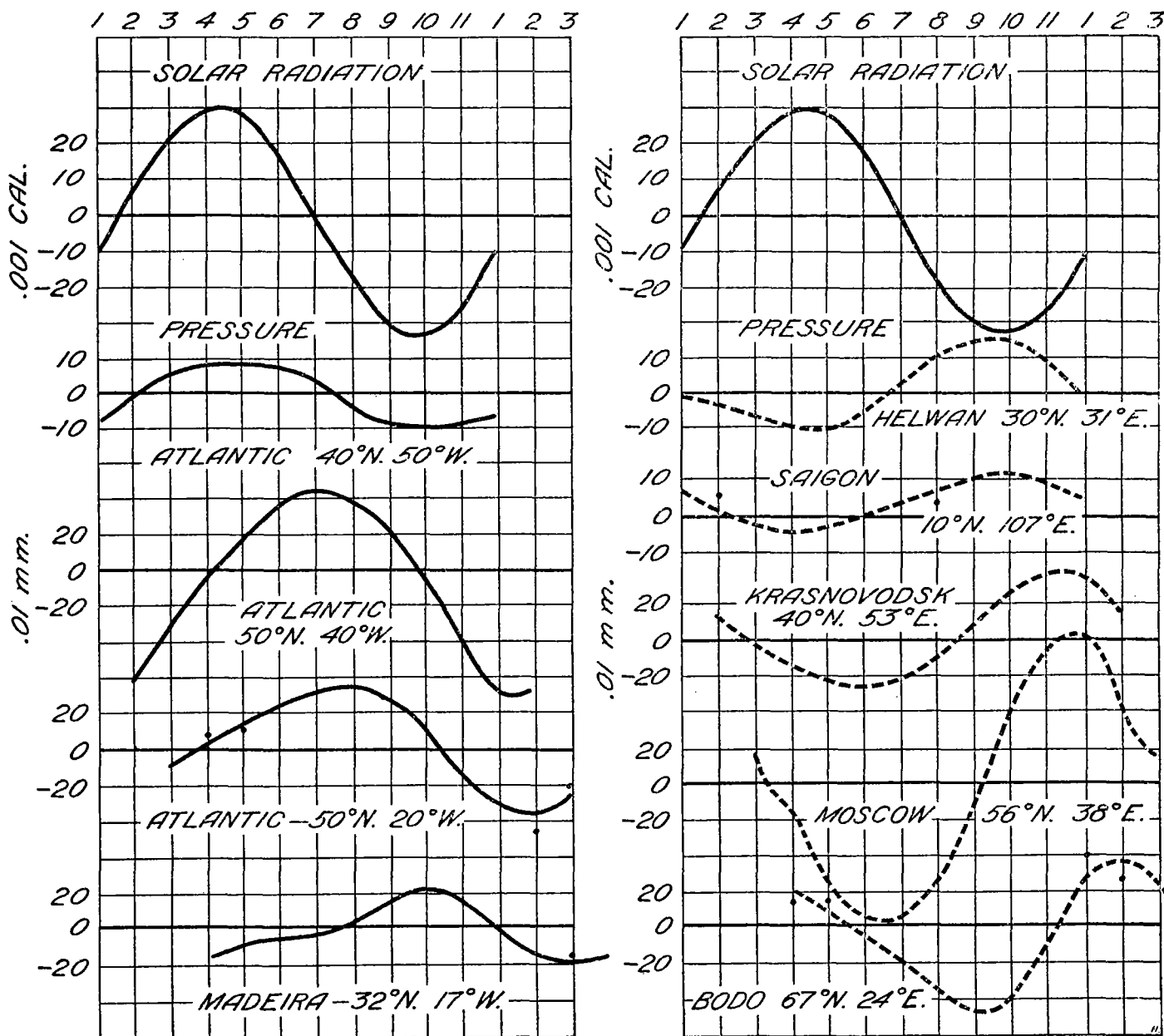


FIGURE 10.—Eleven-month period; means of 10 periods (2).

1932 and filed with the Smithsonian Institution, the receipt of which was noted in a letter from Dr. Abbot on June 2, 1932. This forecast proved to be highly successful and is reproduced in figure 21. The dotted line represents the forecasts made from data preceding 1925, the part after 1931 being projected entirely into the future; the full line connects the observed departures from normal temperature.

Abbot was led by his investigations to conclude that the double sunspot period was more nearly 23 years in length (*Smith. Misc. Coll.* vol. 94, no. 10, 1935), and that the

having at his disposal many centuries of measurements of tree rings which he believes sufficiently represent meteorological changes to permit studies of periodic variations. His latest researches are now being prepared for publication as a memoir by the Carnegie Institution. He has shown exceptional ingenuity in this research, but my own investigations lead me to conclude that it is very difficult to determine periods of any kind accurately from meteorological data owing to frequent changes in phase and intensity of the periods. Such determinations necessarily have a large probable error.

Dinsmore Alter has also studied these changes; but his use of the Schuster periodogram has not been very fruitful in results, because this periodogram assumes a constancy of phase and amplitude in the periods investigated, which apparently does not exist in solar and meteorological periods.

H. W. Clough has also investigated these relations from data covering many centuries, and has concluded that

I find that most of the solar radiation periods so far determined are fractions of the 11.2-year or the 8.4-year periods. The 25-month period which I pointed out in the pressure some 50 years ago is one-fourth of the 8.4-year period; and the 11-month period discussed in this paper is one-twelfth of the 11.2-year sunspot period. The sunspot period varies in amplitude during the 90-year period, being greatest near the maximum of the period.

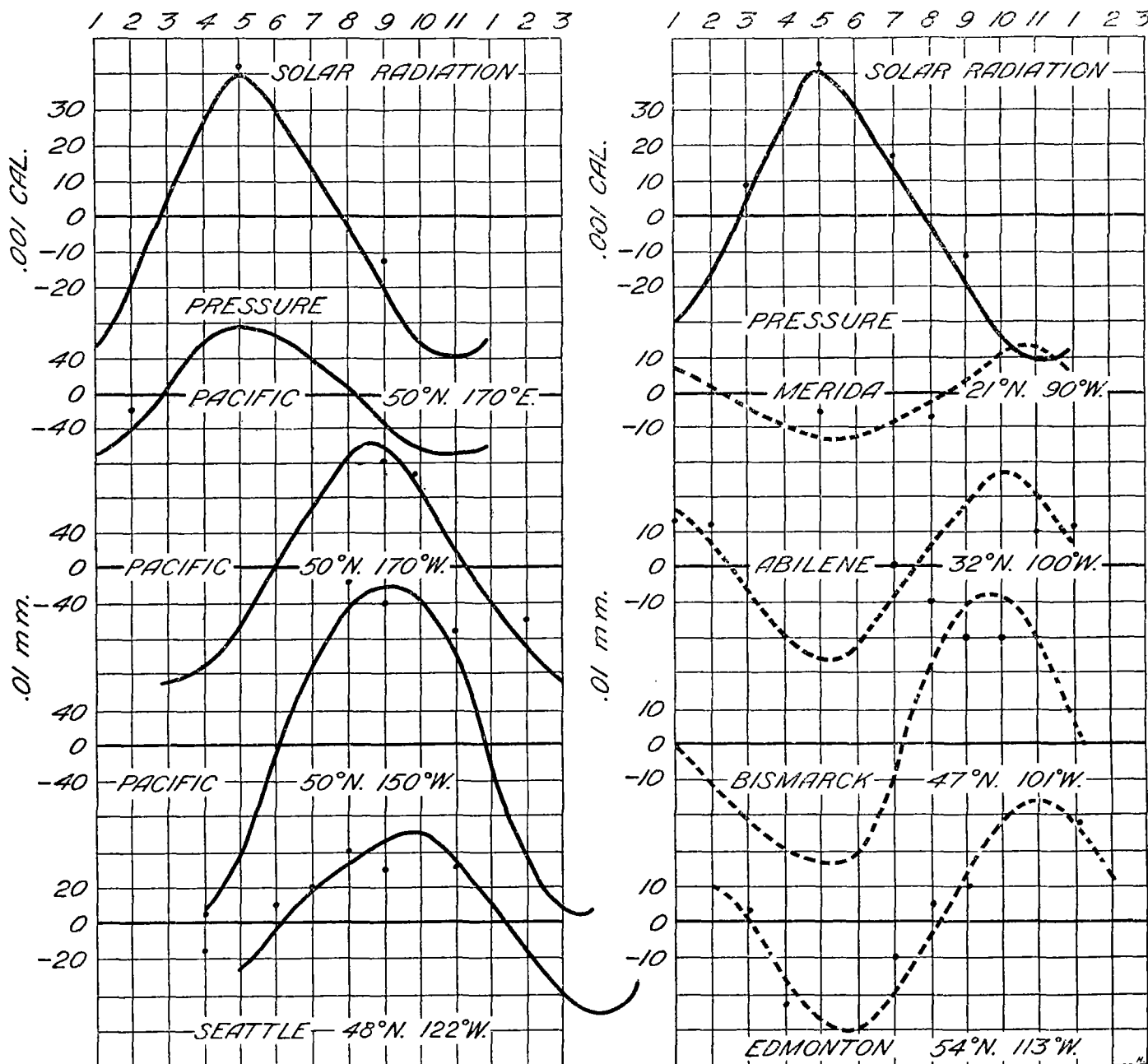


FIGURE 11.—Eleven-month period; means of three periods (1).

solar periods vary systematically in length, going through long cycles of change (MONTHLY WEATHER REVIEW, vol. 61, April 1933, pp. 99-108).

In my latest researches I have found that the sunspot changes during the last 150 years can be closely followed by a combination of four periods. Arranged in the order of importance they are about as follows: 11.2 years (and the half period 5.6 years), 8.4 years, 9.96 years, and about 90 years.

A common multiple of 11.2 and 8.4 is 33.6, and I find that this period conforms so closely with long-period weather changes during the past century that I used it for a general forecast of expected changes many years in advance. This forecast was issued in December 1935 and published in the *Bulletin of the American Meteorological Society* for March 1936. Each subdivision of this 33.6-year period, as for example  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ , etc., is considered a possible solar period.

The double period of sunspots appears also to be a solar period; and no doubt there are longer periods, such as those advocated by A. E. Douglass, Ellsworth Huntington, H. W. Clough, D. Alter, H. P. Gillette, and others in the United States, and by E. Brückner, C. E. P. Brooks, D. Brunt, H. Memery, and others in Europe.

Forecasting variations in amplitude in the solar periods is not yet attained, but it may be that they will be

C. J. Bollinger, of the University of Oklahoma, and A. Jatho and W. Hoxmark, of Buenos Aires, are now at work on such problems. L. Weickman and his pupils, and Bauer, are at work on allied problems in Germany. Julio Bustos Narvarrete is studying the problem in Chile, and Inigo Jones in Australia; also A. N. Wallis in South Africa. G. C. Simpson, Director of the British Meteorological Office has made a profound study of the influence

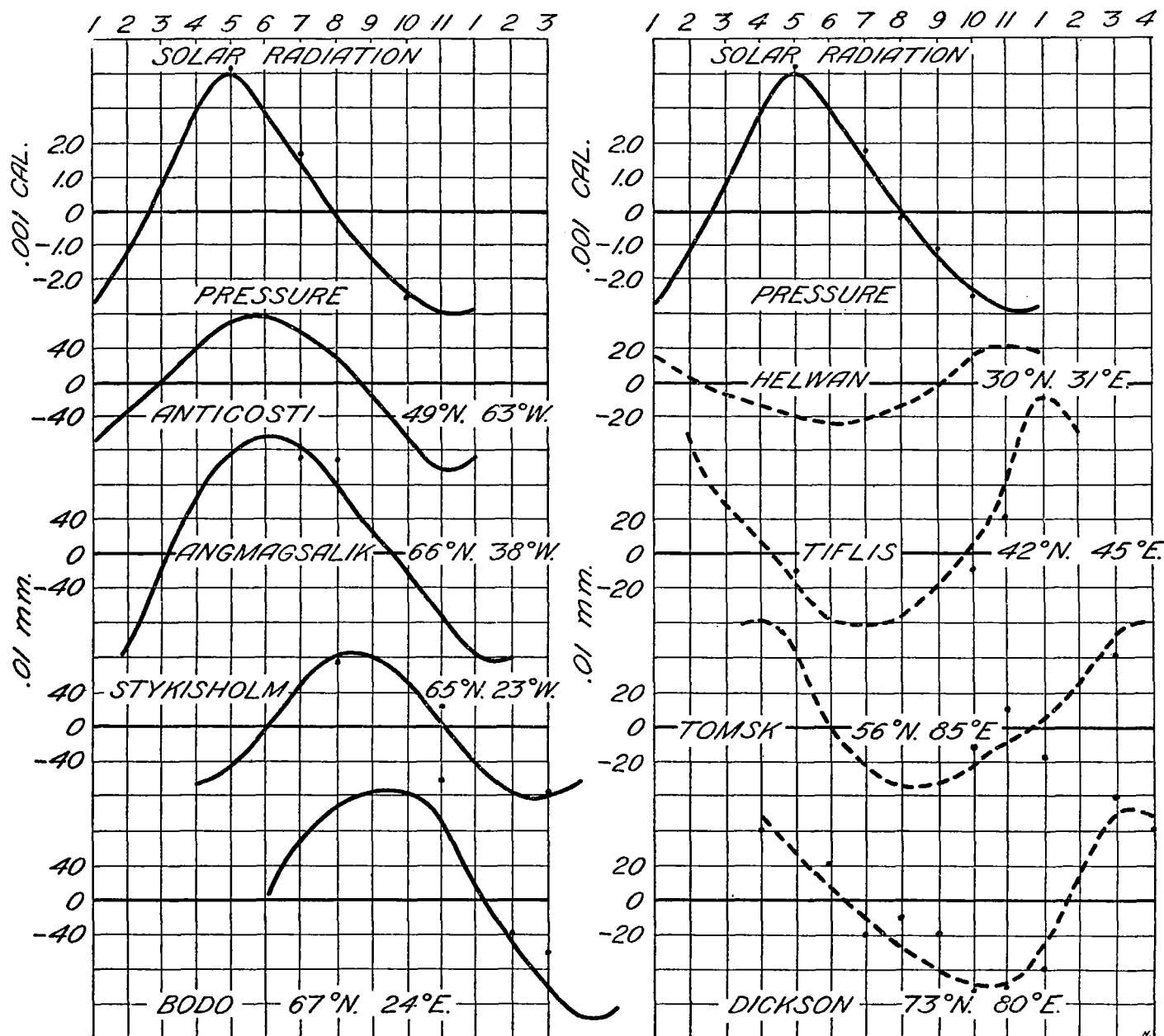


FIGURE 12.—Eleven-month period; means of three periods (2).

found to be cyclical in occurrence and hence predictable. Abbot has been able to anticipate monthly variations in solar radiation fairly well by assuming periods of constant phase and amplitude. I have also made such forecasts that show a correlation with observed values of nearly 0.60, but it seems evident that the solar periods do vary in amplitude and perhaps in phase, so that in accurate forecasting some allowance must be made for these changes. Further work needs to be done in evaluating the seasonal effect on meteorological changes, and the influence of ocean water.

of changes in solar radiation on the heat balance of the atmosphere.

More accurate measures of solar radiation are urgently needed, for it is evident that a change of only a few thousandths of a calory exerts a marked change on our atmosphere. Abbot should be given all the aid possible in this work, and other physicists should be encouraged to devise new methods of attack.

The world owes John A. Roebling a debt of gratitude for the support he has rendered these researches.



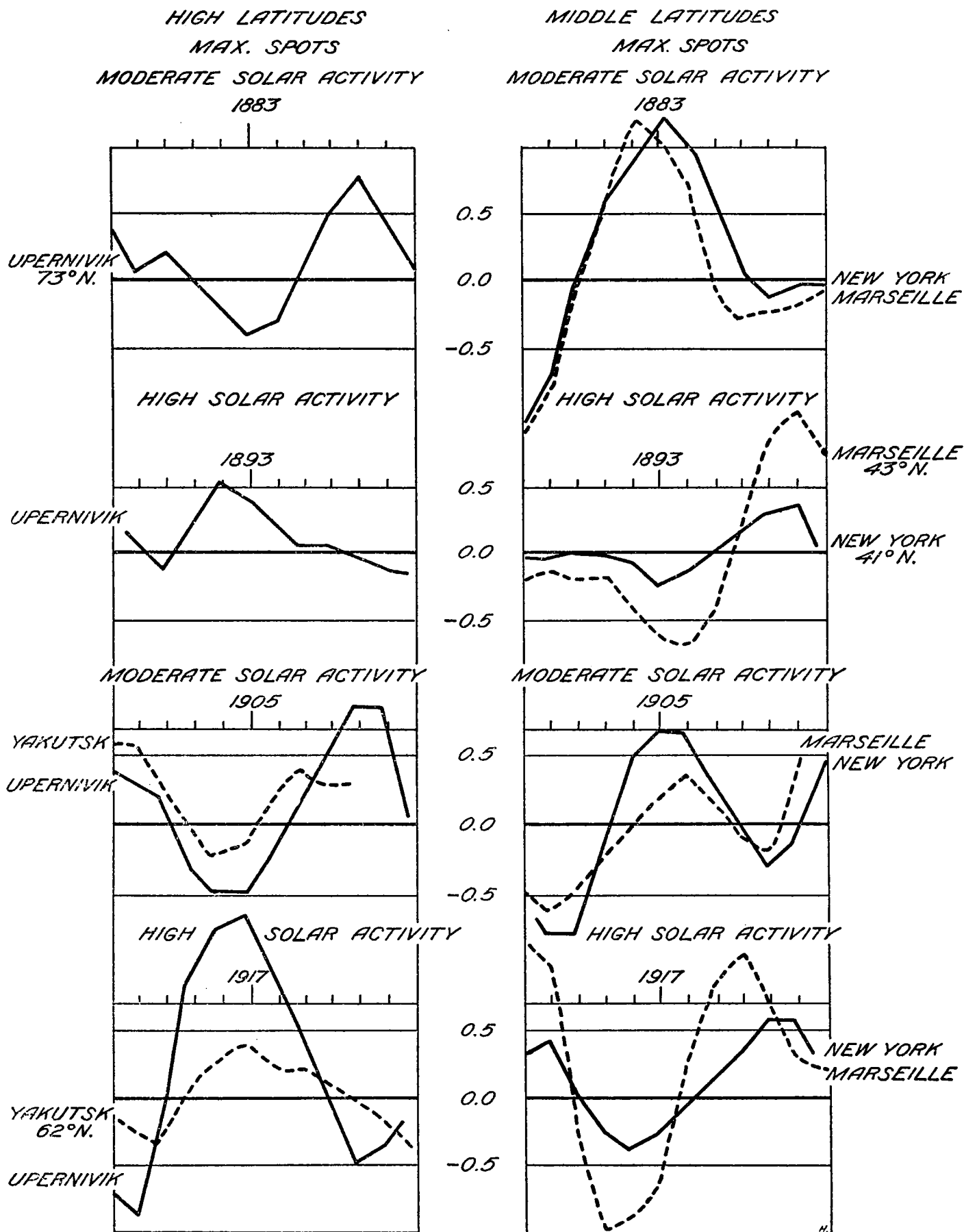


FIGURE 14.—Smoothed annual means of pressure (mm) during the sunspot period, northern hemisphere (*Smith. Misc. Coll.*, vol. 89, no. 15, p. 10).

A third form of smoothing is by means of the harmonic formulas. The classical method of separating periodic oscillations of different lengths from one another for harmonic analysis is to get mean values of each term of the period for a large number of recurrences. In this way if the periodic oscillation is constant in phase and ampli-

The type of harmonic formula used for this purpose is as follows: Let  $l_0, l_1, l_2, l_3, \dots, l_{n-1}$  be observed values which are associated with equidistant values of some argument, say time; then the single periodic terms, namely, coefficients of a sine curve drawn through the observations, may be represented by the trigonometrical formulas:

$$L = A_0 + A_1 \cos \phi + B_1 \sin \phi, \quad (1)$$

in which

$$A_0 = \frac{\sum l}{n}, \quad (2)$$

$$A_1 = \frac{\sum l \cos \phi}{\frac{1}{2}n}, \quad (3)$$

$$B_1 = \frac{\sum l \sin \phi}{\frac{1}{2}n}, \quad (4)$$

$$\frac{A_1}{B_1} = \tan \theta, \quad (5)$$

$$a = \sqrt{A_1^2 + B_1^2} = \frac{A_1}{\sin \theta} \quad (6)$$

$$\phi = \frac{360^\circ}{n}; \quad (7)$$

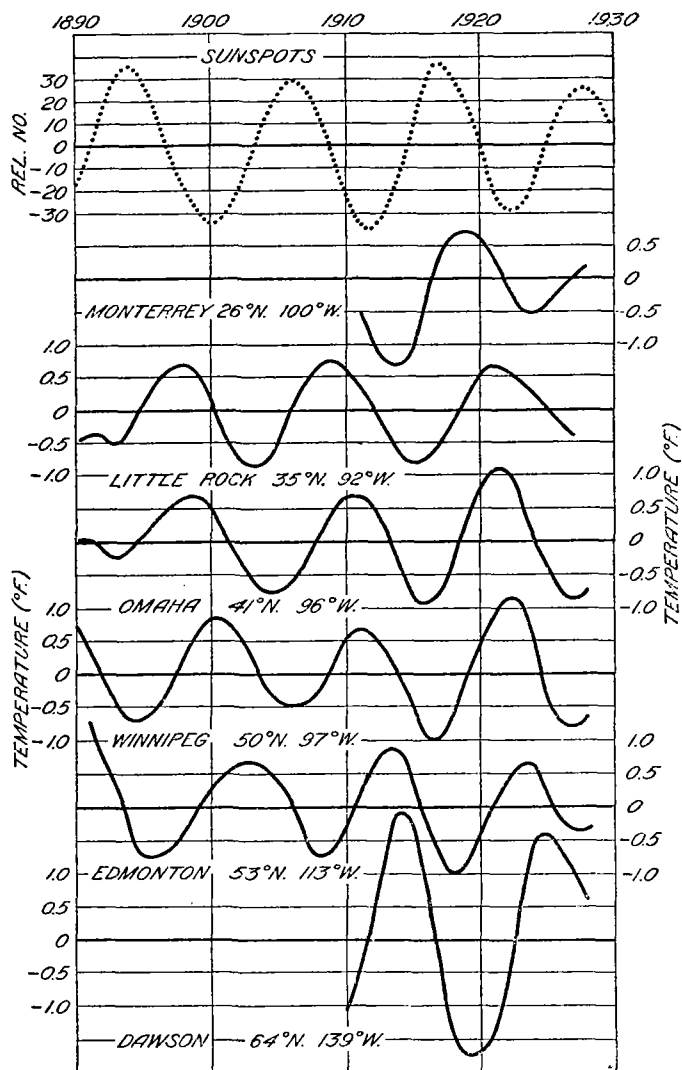


FIGURE 15.—Twelve-year harmonics of sunspots and temperature (1).

tude, all periods of different length and all accidental variations are gradually eliminated by overlapping of positive and negative values. If, for example, the annual period of temperature is desired, the mean values for each month of the year for 50 or 100 years give a smooth curve when plotted, showing the annual period separated from irregularities or from any periodicity of other length.

If, however, a period varies somewhat in length and shows considerable variations in intensity, this method does not apply so well; and in the case of weather periods, which not only vary in length and intensity but show frequent reversals of phase this method of separating periods utterly fails. For this reason the use of the Schuster periodogram is of very questionable value in the search for meteorological periods. The periods cannot be represented in such a case by a set of sine and cosine functions which remain constant in value, but by computing sine and cosine functions for each individual period its oscillations can be followed to some extent and valuable information gained.

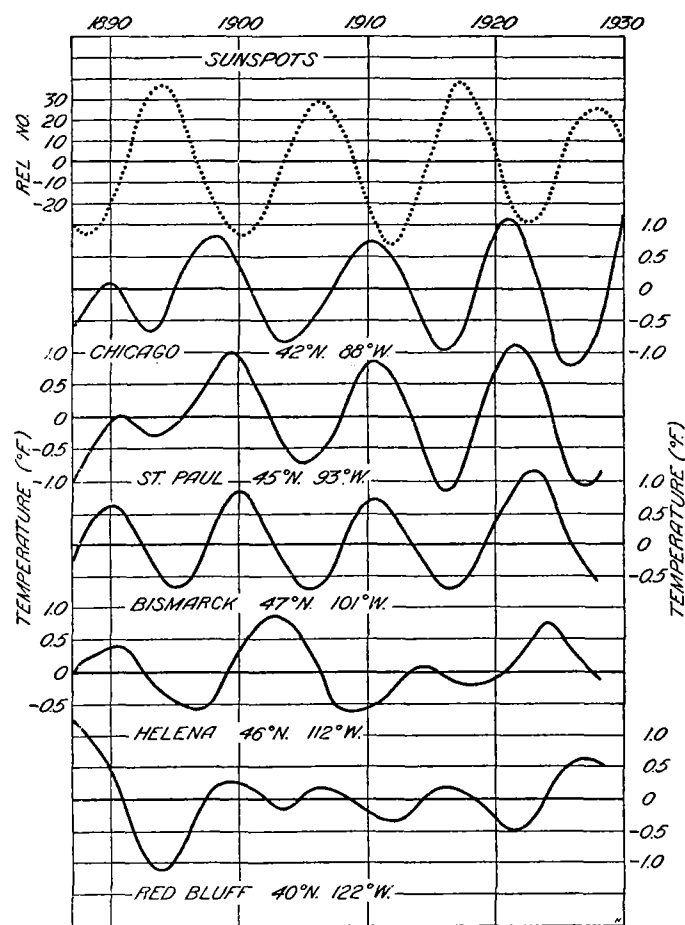


FIGURE 16.—Twelve-year harmonics of sunspots and temperature (2).

where  $\theta$  = angle of the epoch, namely, the angular distance from zero to the part of the sine curve at the beginning of the period, and  $a$  = amplitude, while  $n$  = number of terms used.

The method of computation is shown in table 1. In this table the normal monthly temperatures at New

York, derived from 50 years of observations, are used and the coefficients of a sine curve passing through them are

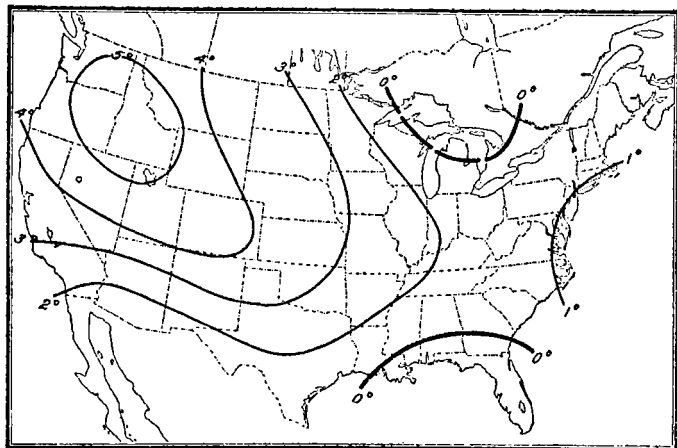


FIGURE 17.—Predicted departures from normal temperature, May 1936.

York, derived from 50 years of observations, are used and the coefficients of a sine curve passing through them are

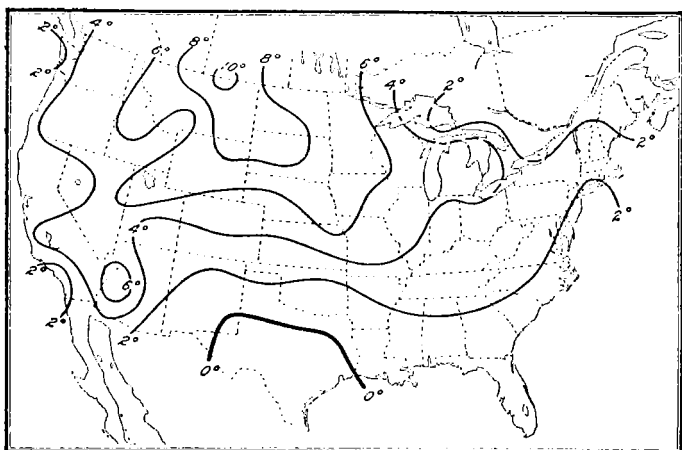


FIGURE 18.—Observed departures from normal temperature, May 1936.

computed. From these coefficients, monthly values are then computed and are given at the bottom of the table. It is seen that these differ very little from the observed

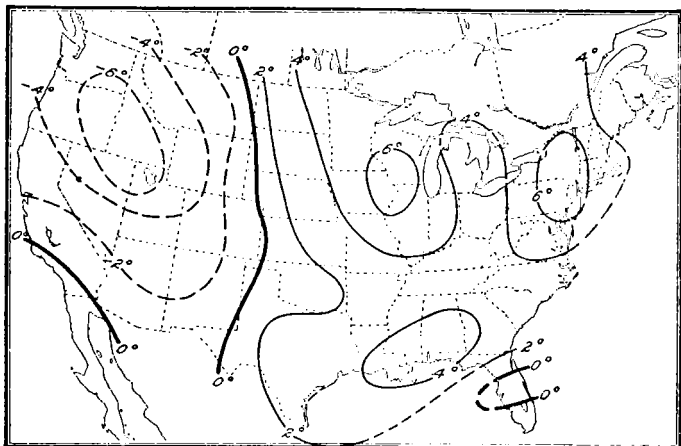


FIGURE 19.—Predicted departures from normal temperature, week ending July 14, 1936.

values, showing that these observed values follow very nearly a sine curve.

The computed values for each month may, however, be obtained in a different way as shown in table 2. In this

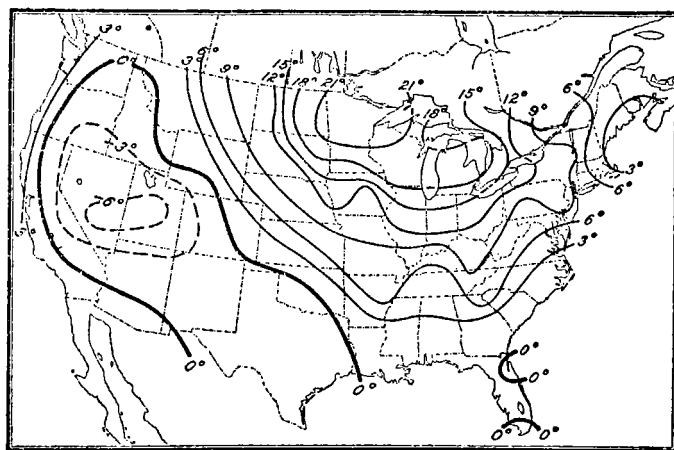


FIGURE 20.—Observed departures from normal temperature, week ending July 14, 1936.

gives the value on a sine curve for the month in which the cosine value is unity.

For example, in the first column of products the cosine is unity in January and the sum of all the products divided by 6 is  $-21.7$ , the same as the computed value in table 1 when  $A_0 = 0$ . In the second column of products

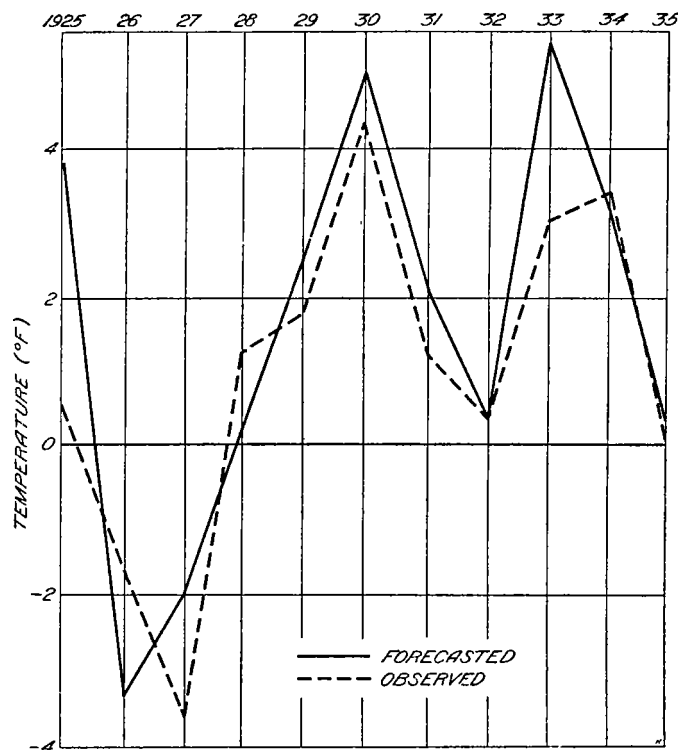


FIGURE 21.—Forecast of departures of June temperatures from normal, New Haven, Conn.

the cosine unity is placed in February, and the sine of the products divided by 6 is  $-20.7$ ; and so on successively for each month. These are nearly identical with the computed values in table 1 when  $A_0 = 0$ . The small differences that exist are accounted for by the fact that the cosine factors were only taken to two or three decimals instead of to four or more. The successive values are thus equivalent to those of moving means except that the smoothing is done by harmonic terms instead of numerically.

This process in reality determines the best fit of a cosine curve with observed values, by the method of correlation. This fact may be seen by the example given in table 3. From this example it is evident that any machine for computing correlation could also be used in getting successive values when smoothing by harmonic values. •

The sine values in table 1 could be used instead of the cosine values, taking the value  $a$  for the months in which the sine values are plus unity and minus unity, respectively.

In table 2 the same numbers repeat themselves after 6 months but with opposite sign. This condition can only happen, however, where the same temperature values are repeated.

In table 4, the observed mean temperatures at New York from 1918 to 1921 are used in the computations. These means are taken in whole degrees (F) in order to simplify computation and the products are arranged along horizontal lines instead of vertical lines as in table 3. Cosine unity is put in the center of the period, and the earlier values are multiplied by values from  $-1.0$  to  $+0.866$  and the later values by  $+1.0$  to  $-0.866$ . In this manner a displacement of the maximum is prevented when the actual oscillation is longer or shorter than the trial period.

In the case of solar and weather periods where the period is changing amplitude and length or even inverting in phase, it is desirable to make the computations cover as short a period of time as possible and to bring the computed value as near the date of the latest observation as possible. This result can be approximated by computing the values for half a period instead of for the whole period.

The method of doing this is shown in table 5. The cosine  $+1.0$  is put opposite the latest observation and the computation is carried backward step by step a half period to cosine  $-1.0$ . The results are placed along a horizontal line and added and the sum placed under "sum  $\frac{1}{2}p$ ." The means in the column headed  $\frac{1}{4}$  give the successive computed values as additional months are added. The final column in this exhibit gives the mean computed from the whole period. The comparison shows that successive values of the half period rarely differed from those of the whole period by more than a few tenths of a degree Fahrenheit. The period covered by the calculation is longer than a half period since it runs from cosine  $+1.0$  to  $-1.0$ . Hence it embraces one term of full value more than a half period, and therefore the divisor is  $\frac{1}{2}h + 1$ .

When the oscillations vary in length and amplitude this method of computing by a half period instead of a whole period does not entirely eliminate irregularities in the data, nor completely separate periods of different length. It is being used tentatively. For studies of periodicity and for forecasting, chief reliance is based on the smoothing by whole periods illustrated in table 6.

Table 6 shows this plan of smoothing as applied to daily observations of pressure at New York from August 23 to September 23, 1930. The smoothing is done for trial periods of 8, 10, and 12 days. The final results are shown in the column headed "Means."

This smoothing can be done mechanically as well as numerically. In the late spring of 1930 Maj. Lawrence Clayton, of the United States Army, then on leave, made a search, at my request, for mechanical methods of harmonic smoothing. He found that Vanevar Bush, of the Massachusetts Institute of Technology, had devised a machine, for computing correlations, called a photoelectric integrator which could be adapted to perform all of the various processes of smoothing. The machine was then in a crude form and I did not have the means to improve or modify it, so continued to smooth by a simplified process of numerical computations. This machine is now being im-

proved by Bush and can no doubt greatly simplify the work of the various smoothing processes.

In order to test whether the irregularities in the length and intensity of the pulsations shown by the analyzed data are due entirely to irregularities in the position and movements of the *meiopleions* or whether they may be due to combinations of periods near the same length, the values such as shown in the column of means in table 6 are further analyzed by averaging them in periods of different length.

For example, there is some evidence in table 6 of solar periods of about 7 and of about 9 days (one-fourth and one-third of a solar rotation). To test this, the *harmonics* for 8 days shown in the column of means of the 8-day analysis were averaged for overlapping periods of 7 and 9 days. The 10-day harmonics and the 12-day harmonics were averaged in periods of 9 days. The overlapping means of three periods illustrated in table 7 show that both a 7-day and a 9-day period can be detected in the means, the 9-day period having the larger amplitude.

In forecasting, generally only the periods having the larger amplitudes at any given time are used. In case the phase of the period does not invert frequently, as in the case of the solar periods, then overlapping means of 5 and 10 periods may be obtained advantageously.

#### SUMMARY

My method of forecasting consists of—

(1) The analysis of weather phenomena into pulses, waves, or periods of different length; this is done for each station in a network of stations, and the latest values plotted on maps. From a succession of such maps the movements of areas of excess and deficiency can be followed and their future position anticipated.

(2) Correlation of the meteorological pulses or waves with solar radiation pulses or waves found in the same way.

(3) Projection of these waves ahead into the future, using for this purpose the mean lengths of the solar periods determined by experience or calculation.

(4) Reading from each of the curves so projected ahead, the value for some particular time or epoch desired, and summing the different values thus obtained.

(5) The process described in (3) is done for a network of stations, the sums are plotted on maps and lines of equal value drawn. These maps then become a forecast for the area covered (see figs. 17 and 19).

TABLE 1.—Example computation by harmonic formula

Cycle of 360° divided into 12 parts	Sine values	Cosine values	Normal monthly temper- atures, New York <sup>1</sup>		Temperatures—	
					By sine values	By cosine values
					(1)	(2)
(1)	(2)	(3)	(1)	(5)	(6)	(7)
				° F.	° F.	° F.
0°	0.00	1.00	January	30.6	0.0	30.6
30°	0.50	0.866	February	30.5	15.3	26.4
60°	0.866	0.50	March	38.0	32.9	19.0
90°	1.00	0.00	April	48.5	48.5	0.0
120°	0.866	-0.50	May	59.4	51.4	-29.7
150°	0.50	-0.866	June	68.5	34.3	-59.3
180°	0.00	-1.00	July	73.5	0.0	-73.5
210°	-0.50	-0.866	August	72.1	-36.1	-62.4
240°	-0.866	-0.50	September	66.4	-57.5	-33.2
270°	-1.00	-0.00	October	55.8	-55.8	0.0
300°	-0.866	0.50	November	44.1	-38.2	22.1
330°	-0.50	0.866	December	34.3	-17.2	29.7
Sum					-22.4	-130.3

<sup>1</sup> Mean of 51 years, 1873-1923.

$a = \frac{1}{12} \sqrt{(22.4)^2 + (130.3)^2} = 22.04$ ;  $\tan \theta = \frac{130.3}{22.4} = 5.81$ ;  $\theta = 260^\circ$ .

$\theta$  = epoch;  $a$  = amplitude;  $A_0 = 51.8$  = Mean for year.

#### Monthly values computed from $\theta$ and $a$

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
For $A_0 = 0$ .....	-21.7	-20.7	-14.1	-3.8	7.6	17.0	21.7	20.7	14.1	3.8	-7.6	-17.0
For $A_0 = 51.8$ .....	30.1	31.1	37.7	48.0	59.4	68.8	73.5	72.5	65.9	55.6	44.2	34.8

TABLE 2.—Temperatures multiplied by cosine values

	Normal monthly tempera- tures at New York <sup>1</sup>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
January	30.6	30.6											
February	30.5	26.4	30.5										
March	38.0	19.0	32.9	38.0									
April	48.5	0.0	24.3	42.0	48.5								
May	59.4	-29.7	0.0	29.7	51.4	59.4							
June	68.5	-59.3	-34.3	0.0	34.3	59.3	68.5						
July	73.5	-73.5	-63.7	-36.8	0.0	36.8	63.7	73.5					
August	72.1	-62.4	-72.1	-62.4	-36.1	0.0	36.1	62.4	72.1				
September	66.4	-33.2	-57.5	-66.4	-57.5	-33.2	0.0	33.2	57.5	66.4			
October	55.8	0.0	-27.9	-48.3	-55.8	-48.3	-27.9	0.0	27.9	48.3	55.8		
November	44.1	22.1	0.0	-22.1	-38.2	-44.1	-38.2	-22.1	0.0	22.1	38.2	44.1	
December	34.3	29.7	17.2	0.0	-17.2	-29.7	-34.3	-29.7	-17.2	0.0	17.2	29.7	34.3
January	30.6		26.5	15.3	0.0	-15.3	-26.5	-30.6	-26.5	-15.3	0.0	15.3	26.5
February	30.5			15.3	0.0	-15.3	-26.4	-30.5	-26.4	-15.3	0.0	15.3	26.4
March	38.0			32.9	19.0	0.0	-19.0	-38.0	-32.9	-19.0	0.0	19.0	32.9
April	48.5				42.0	24.3	0.0	-24.3	-42.0	-48.5	-42.0	-24.3	42.0
May	59.4					51.4	29.7	0.0	-29.7	-51.4	-48.5	-42.0	48.5
June	68.5						59.3	34.3	0.0	-34.3	-59.3	-63.7	63.7
July	73.5							63.7	62.4	36.1	0.0	-36.1	62.4
August	72.1									57.5	33.2	0.0	57.5
September	66.4										48.3	27.9	0.0
October	55.8											38.2	27.9
November	44.1												38.2
December	34.3												
Sums		-130.3	-124.3	-84.6	-22.4	45.9	101.8	130.3	124.2	84.6	22.3	-45.9	-101.8
Means, $\frac{1}{12}$		-21.7	-20.7	-14.1	-3.7	7.6	17.0	21.7	20.7	14.1	3.7	-7.6	-17.0
Observed		-21.2	-21.3	-13.8	-3.3	7.6	16.7	21.7	20.3	14.6	4.0	-7.7	-17.5

<sup>1</sup> Averages of 51 years, 1873-1923.

TABLE 3.—Harmonic terms computed by correlations

(1)	(2)	(3)	(4)	(5)	(6)
Month	$x = \text{cosine values}$	$y = \text{temper-ature de-partures}$	$xy = \text{prod-uct}$	$x^2$	$y^2$
July	1.00	21.7	21.7	1.00	471
August	.866	20.3	17.6	.75	412
September	.50	14.6	7.3	.25	213
October	.00	4.0	.0	.00	16
November	-.50	-7.7	-3.8	.25	136
December	-.866	-17.5	-15.1	.75	306
January	-1.00	-21.2	-21.2	1.00	449
February	-.866	-21.3	18.4	.75	454
March	-.50	-13.8	6.9	.25	190
April	.00	-3.3	.0	.00	44
May	.50	7.6	3.8	.25	134
June	.866	16.7	14.5	.75	279
Sums	.00	.0	130.3	6.00	3104

$$\text{Correlation coefficient } r = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}} = \frac{130.3}{\sqrt{6 \times 3104} \sqrt{136.8}} = 0.95$$

$$a = \frac{\sum xy}{\sum x^2} = \frac{130.3}{6} = 21.7$$

NOTE.— $a$  = ratio of the observed values to a cosine series having plus unity in July and minus unity in January.

TABLE 4.—Monthly mean temperatures at New York smoothed harmonically (1)

Angle	180°	210°	240°	270°	300°	330°	0°	30°	60°	90°	120°	150°	Sum	$\frac{1}{12}$
Cosine	-1.0	-0.866	-0.5	0	0.5	0.866	1.0	0.866	0.5	0	-0.5	-0.866		
1918														
Jan	21													
Feb	29													
Mar	41													
Apr	49													
May	63													
June	66													
July	72	-21	-25	-21	0	32	57	72	64	31	0	-23	-33	22.2

TABLE 4.—Monthly mean temperatures at New York smoothed harmonically (1)—Continued

Angle	180°	210°	240°	270°	300°	330°	0°	30°	60°	90°	120°	150°	Sum	$\frac{1}{12}$
Cosine	-1.0	-0.866	-0.5	0	0.5	0.866	1.0	0.866	0.5	0	-0.5	-0.866		
1918														
Aug	74	-29	-35	-25	0	33	62	74	53	29	0	-20	-30	112
Sept	62	-41	-42	-32	0	36	64	62	51	23	0	-18	-30	73
Oct	58	-49	-54	-33	0	37	53	58	39	20	0	-18	-36	17
Nov	46	-63	-57	-36	0	31	51	46	33	18	0	-21	-41	39
Dec	39	-66	-62	-37	0	29	39	39	30	18	0	-24	-51	-85
1919														
Jan	35	-72	-64	-31	0	23	33	35	30	21	0	-30	-59	-114
Feb	35	-74	-53	-29	0	20	30	35	36	24	0	-35	-63	-109
Mar	42	-62	-50	-23	0	18	30	42	41	30	0	-37	-60	-71
Apr	48	-58	-39	-20	0	18	36	48	51	35	0	-35	-57	-21
May	60	-46	-33	-18	0	21	41	60	59	37	0	-33	-50	38
June	69	-39	-30	-15	0	24	51	69	63	35	0	-29	-38	88
July	73	-35	-30	-21	0	30	59	73	60	33	0	-22	-26	121
Aug	70	-35	-36	-24	0	35	63	70	57	29	0	-15	-21	123
Sept	66	-42	-41	-30	0	37	60	66	50	22	0	-12	-25	85
Oct	58	-48	-51	-35	0	35	57	58	38	15	0	-15	-34	20
Nov	44	-60	-59	-37	0	33	50	44	26	12	0	-20	-40	-51
Dec	30	-69	-63	-35	0	29	38	30	21	15	0	-24	-49	-107
1920														
Jan	24	-73	-60	-33	0	22	26	24	25	20	0	-26	-58	-136
Feb	29	-70	-57	-29	0	15	21	29	34	24	0	-34	-63	-130
Mar	40	-66	-50	-22	0	12	25	40	40	29	0	-36	-63	-91
Apr	47	-58	-38	-15	0	15	34	47	49	34	0	-36	-58	-26
May	57	-44	-26	-12	0	20	40	57	58	36	0	-34	-51	44
June	67	-30	-21	-15	0	24	49	67	63	36	0	-30	-38	105
July	72	-24	-25	-20	0	29	58	72	63	34	0	-22	-33	132
Aug	72	-29	-35	-24	0	34	63	72	58	30	0	-19	-28	122
Sept	67	-40	-40	-29	0	36	63	67	51	22	0	-17	-30	83
Oct	60	-47	-49	-34	0	36	58	60	38	19	0	-18	-41	22
Nov	44	-57	-58	-36	0	34	51	44	23	17	0	-24	-47	-43
Dec	38	-67	-63	-36	0	30	38	38	28	18	0	-28	-52	-94
1921														
Jan	33	-72	-63	-34	0	22	33	33	30	24	0	-30	-60	-117
Feb	35	-72	-58	-30	0	19	28	35	41	28	0	-35	-65	-109
Mar	48	-67	-51	-22	0	17	30	48	47	30	0	-38	-60	-91
Apr	55	-60	-38	-19	0	18	41	55	52	35	0	-35	-55	-26
May	60	-44	-33	-17	0	24	47	60	60	38	0	-33	-50	44
June	70	-38	-29	-18	0	28	52	70	65	38	0	-28	-38	105
July	76	-33	-30	-24	0	30	60	76	60	38	0	-24	-33	132

TABLE 5.—Monthly mean temperatures at New York smoothed harmonically (2)

Angle...	0°	30°	60°	90°	120°	150°	180°	Sum 1/2 period	Mean 1/4	Sum less column 0°	Sum whole period	Mean 1/6
Cosine...	1.0	0.866	0.5	0	-0.5	-0.866	-1.0					
<b>1918</b>												
Jan.	21											
Feb.	29											
Mar.	41											
Apr.	49											
May	63											
June	66											
July	72	57	32	0	-21	-25	-21	94	23.6	22	133	22.2
Aug.	74	62	33	0	-25	-35	-29	80	20.0	6	112	18.7
Sept.	62	64	36	0	-32	-42	-41	47	11.8	-15	72	12.0
Oct.	58	53	37	0	-33	-54	-49	12	3.0	-46	17	2.9
Nov.	46	50	31	0	-36	-57	-63	-29	-7.2	-75	-40	-6.7
Dec.	39	39	29	0	-37	-62	-66	-58	-14.5	-97	-85	-14.2
<b>1919</b>												
Jan.	35	33	23	0	-31	-64	-72	-76	-19.0	-111	-114	-19.0
Feb.	35	30	20	0	-29	-53	-74	-71	-17.8	-106	-109	-18.2
Mar.	42	30	18	0	-23	-50	-62	-45	-11.2	-87	-71	-11.8
Apr.	48	36	18	0	-20	-39	-58	-15	-8.5	-63	-21	-8.5
May	60	41	21	0	-18	-33	-46	25	6.3	-35	38	6.3
June	69	51	24	0	-18	-30	-39	57	14.3	-12	88	14.7
July	73	59	30	0	-21	-30	-35	76	19.0	3	121	20.5
Aug.	70	63	35	0	-24	-36	-35	73	18.3	3	123	20.5
Sept.	66	60	37	0	-30	-41	-42	50	12.5	-16	85	14.2

<sup>1</sup> Add sum in preceding column to sum 6 months later with sign reversed.

TABLE 5.—Monthly mean temperatures at New York smoothed harmonically (2)—Continued

Angle...	0°	30°	60°	90°	120°	150°	180°	Sum 1/2 period	Mean 1/4	Sum less column 0°	Sum whole period	Mean 1/6
Cosine...	1.0	0.866	0.5	0	-0.5	-0.866	-1.0					
<b>1919</b>												
Oct.	58	57	35	0	-35	-51	-48	16	4.0	-42	20	5.3
Nov.	44	50	33	0	-37	-59	-60	-29	-7.3	-73	-51	-8.5
Dec.	30	38	29	0	-35	-63	-69	-70	-17.5	-100	-107	-17.6
<b>1920</b>												
Jan.	24	26	22	0	-33	-60	-73	-94	-23.5	-118	-136	-22.7
Feb.	29	21	15	0	-29	-57	-70	-91	-22.8	-120	-129	-21.7
Mar.	40	25	12	0	-22	-50	-66	-61	-15.2	-101	-91	-15.2
Apr.	47	34	15	0	-15	-38	-58	-15	-8.8	-62	-26	-8.8
May	57	40	20	0	-12	-26	-44	35	8.7	-22	44	7.3
June	67	49	24	0	-15	-21	-30	74	18.5	7	105	17.5
July	72	58	29	0	-20	-25	-24	90	22.5	18	132	22.0
Aug.	72	63	34	0	-24	-35	-29	81	20.2	9	122	20.3
Sept.	67	63	36	0	-29	-40	-40	57	14.2	-10	83	13.8
Oct.	60	58	36	0	-34	-49	-47	21	6.0	-36	22	5.7
Nov.	44	51	34	0	-36	-58	-57	-22	-5.5	-66	-43	-7.2
Dec.	38	38	30	0	-36	-62	-67	-59	-14.7	-97	-93	-15.7
<b>1921</b>												
Jan.	33	33	22	0	-34	-62	-72	-80	-20.0	-113	-117	-19.5
Feb.	35	28	19	0	-30	-58	-72	-78	-19.5	-113	-113	-19.5
Mar.	48	30	17	0	-22	-51	-67	-45	-11.2	-93	-73	-11.2
Apr.	55	41	18	0	-19	-38	-60	-3	-0.8	-58	-33	-0.8
May	60	47	24	0	-17	-33	-44	37	9.3	-23	-3	9.3
June	70	52	28	0	-18	-28	-38	66	16.5	-4	100	16.5
July	76	60	30	0	-24	-30	-33	79	19.8	4	133	20.5

TABLE 6.—8 a. m. pressures at New York analyzed in periods of 8, 10, and 12 days

	Observed pressure 29.00 in.	8-day analysis							10-day analysis							12-day analysis									
		1.0	0.7	0	-0.7	Sum	Differ- ence <sup>1</sup> 4	1/4	1.0	0.8	0.3	0.3	-0.8	Sum	Differ- ence <sup>1</sup> 5	1/5	1.0	0.866	0.5	0	-0.5	-0.866	Sum	Differ- ence <sup>1</sup> 6	1/6
1930																									
Aug.	13	+																							
	14	1.34	134						134								134								
	15	1.12	112						112								112								
	16	.96	96						96								96								
	17	.88	88						88								88								
	18	.98	98	62	0	-78	82	-49	1-12	98							96								
	19	.82	82	69	0	-67	84	-34	-9	82	78	26	-29	-90	67	-24	-5	82							
	20	.94	94	57	0	-62	89	11	3	94	68	29	-26	-77	88	7	1	94	71	49	0	-48	-97	69	-28
	21	1.03	108	66	0	-69	105	23	6	108	75	25	-29	-70	109	-8	-2	108	83	41	0	-44	-82	106	-8
	22	1.12	112	76	0	-57	131	18	5	112	86	28	-25	-78	123	-4	-1	112	93	47	0	-49	-75	128	1
Sept.	23	1.06	106	78	0	-66	118	-19	-5	106	90	32	-28	-68	132	-2	0	106	97	54	0	-41	-83	133	4
	24	.80	80	74	0	-76	78	-31	2-8	80	85	33	-32	-75	91	-12	-2	80	92	56	0	-47	-71	110	20
	25	1.04	104	56	0	-78	82	-14	-4	104	64	32	-32	-86	81	-2	0	104	69	53	0	-54	-83	89	6
	26	1.16	116	71	0	-74	113	22	6	116	83	24	-32	-90	101	10	2	116	90	40	0	-56	-93	97	-20
	27	1.12	112	81	0	-56	137	38	10	112	93	31	-24	-85	127	1	0	112	100	52	0	-53	-97	114	-38
	28	1.04	104	78	0	-73	109	-19	-5	104	90	35	-31	-64	134	-9	-2	104	97	58	0	-49	-92	127	-27
	29	1.04	104	73	0	-81	96	-59	2-15	104	83	34	-35	-83	103	-47	-9	104	96	56	0	-52	-69	129	7
	30	.96	96	73	0	-78	91	-42	-11	96	83	31	-34	-93	83	-19	-4	96	90	52	0	-58	-90	90	-5
	31	1.04	104	67	0	-73	98	16	4	104	77	31	-31	-90	91	13	3	104	83	52	0	-58	-100	83	0
	1	1.28	128	73	0	-73	128	44	11	128	83	29	-31	-83	126	32	6	128	90	48	0	-52	-97	117	34
2	1.32	132	90	0	-67	155	41	10	132	102	31	-39	-83	143	37	7	132	110	52	0	-52	-90	152	56	
3	1.14	114	92	0	-73	133	18	5	114	106	38	-31	-77	150	47	9	114	114	64	0	-48	-90	154	64	
4	.92	92	80	0	-90	82	-7	2-2	92	91	40	-28	-83	102	28	6	92	99	66	0	-52	-83	122	32	
5	1.12	112	64	0	-92	84	4	1	112	74	34	-40	-102	78	-19	-4	112	80	57	0	-64	-90	95	-16	
6	1.16	116	78	0	-80	114	3	1	116	90	28	-34	-106	94	-34	-7	116	97	46	0	-66	-110	83	-54	
7	.98	98	81	0	-64	115	-11	-3	98	93	34	-28	-91	106	-38	-8	98	100	56	0	-57	-114	83	-44	
8	.98	98	69	0	-79	80	-41	-10	98	78	35	-34	-74	103	-15	-3	98	85	58	0	-46	-99	96	-4	

<sup>1</sup> Differences from sums half a period later.

<sup>2</sup> Minima.

TABLE 7.—Averaging harmon by solar periods

		8-day harmon of pressure at New York																	
7-day period		1	2	3	4	5	6	7		1	2	3	4	5	6	7			
1930																			
Aug. 18		-9	3	6	5	-5	-8	-4		Means of 3									
Aug. 25		6	10	-5	-15	-11	4	11		2	6	0	-3	1-5	-2	-1			
Sept. 1		10	5	-2	1	1	-3	-10		4	7	2	0	1-2	0	-3			
Sept. 8		-3	7	13	13	3	-1	-11		-3	0	3	7	4					
Sept. 15		-14	-11	-2	6	7													
etc.																			
9-day period		1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1930																			
Aug. 18		1-9	3	6	5	-5	-8	-4	6	10	Means of 3								
Aug. 27		-5	1-15	-11	4	11	10	5	-2	1	-4	1-5	-5	2	4	5	4	2	3
Sept. 5		1	-3	1-10	-3	7	13	13	3	-1	-5	1-11	-11	0	8	10			
Sept. 14		-11	1-14	-11	-2	6	7												
etc.																			
		10-day harmon of pressure at New York																	
9-day period		1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1930																			
Aug. 18		1-5	2	2	-1	0	-2	0	2	0	Means of 3								
Aug. 27		0	1-9	-4	3	6	5	11	6	-1	1-5	-1	-2	0	4	5	7	5	-3
Sept. 5		1-9	-6	-5	-2	5	12	11	7	-4	-7	1-10	-6	-1	3				
Sept. 14		-12	1-14	-9	-3	-1													
etc.																			